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RED SUPERGIANTS AND NEUTRINO EMISSION

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ABSTRACT

Data on the supergiants in all early-type clusters and associations of the Galaxy and Magellanic Clouds which contain known red-supergiant members have been collected and analyzed in the light of current theory. No major differences seem to exist between the Galaxy and the Magellanic Clouds in this study.

The composite bolometric H-R diagram for the "M associations" shows (1) many blue supergiants (O9.5–A5) mostly in the range B0–B3 with $M_{\rm bol} \sim -6$ to -10; (2) a roughly equal number of red supergiants (M0–M4.5) mostly in the range M0–M2, but the brightest being latest in spectral type, with $M_{\rm bol} \sim -5$ to -8 and no correlation with luminosity class; and (3) a thin scatter of yellow supergiants (F, G, and K).

Dating of the clusters from the luminosities of their blue supergiants indicates decreasing and increasing numbers of blue and red supergiants, respectively, with age. Luminosity functions show that the blue/red ratio is >5 at high masses (15–60 $\mathfrak{M}_\odot$) and ~ 1 at lower masses (10–15 $\mathfrak{M}_\odot$). While a one-to-one correspondence exists between luminosity and mass for the core helium-burning blue supergiants, it is lacking for the red supergiants because of a "funneling effect" (in analogy with the K giants). The red supergiants are shown theoretically to be in two basically different evolutionary phases: (1) helium core contraction and early core helium burning and (2) carbon core contraction and later phases The first case explains the blue/red ratio at low masses. The second case refers to high masses and depends on the existence of the hypothetical photoneutrino process; the predicted blue/red ratio is <17 and <1.7 with and without the neutrino process, respectively.

On the basis of the large blue/red ratio among the most massive supergiants and the independent theoretical and observational evidence that early core helium burning occurs in the red-supergiant region, it is concluded that some effect must take massive stars rapidly through, or away from, the red-supergiant region after core helium burning. Mixing and mass loss are shown to be inadequate mechanisms. If photoneutrino losses are permitted to occur, the stars do evolve rapidly as red supergiants, as required by the observations. One might accept this as supporting evidence for the existence of a direct electron-neutrino weak interaction in nature.

I. INTRODUCTION

The red supergiants have been an astronomical puzzle for more than half a century. Following the pioneer work of Miss Maury and Hertzsprung, who established the high luminosity of these stars, various attempts have been made to fit them observationally and theoretically into the scheme of stellar evolution. Originally, an idea of Lockyer's led to the proposal by Russell and Hertzsprung that the M supergiants are in an early stage of pre-main-sequence contraction. This proposal was later adopted by Shapley, who obtained the Hertzsprung-Russell (H-R) diagrams of a number of young star associations apparently containing M supergiants. Much later, an alternative notion was put forward by Deutsch, who found evidence among the distribution of stars in the general field that the M supergiants are more likely to be relatively old derivatives of main-sequence O and B stars. Such observational evidence was reinforced by the semi-empirical evolution tracks of Sandage and, later, by the theoretical evolution tracks of Hayashi and Cameron.

The most extensive effort has been applied toward explaining the halo of red supergiants which surrounds the double cluster h and χ Per and is apparently coincident with the distribution of blue supergiants in the area. According to a theoretical model sequence due to Hayashi and his associates (Hayashi and Cameron 1962*a;* Hayashi, Hōshi, and Sugimoto 1962), the blue supergiants are in the core helium-burning phase of evolution and the red supergiants are in carbon core contraction and later phases.

Since the lifetime of the later phases is drastically shortened by the inclusion of certain hypothetical neutrino processes in the models, Hayashi concluded from the comparable numbers of blue and red supergiants that such neutrino processes may be forbidden. The neutrino process of greatest importance in this context is the photoneutrino process (Chiu and Stabler 1961; Ritus 1961), while of slightly lesser importance are the pair-annihilation process (Chiu and Morrison 1960; Chiu 1961*b*) and the plasma-neutrino process (Adams, Ruderman, and Woo 1963; Zaidi 1965). All three processes depend on a postulated direct interaction between electrons and neutrinos as provided by the current-current interaction hypothesis of Feynman and Gell-Mann.

Other astrophysical evidence, however, indicates not only that the electron-neutrino interaction probably exists in nature, but that it exists in the strength suggested by current theory. This evidence includes: (1) lifetime of the ultraviolet dwarfs (Chin, Chiu, and Stothers 1966*a*, *b;* Stothers 1966*c*, 1968), (2) lifetime of the central stars of planetary nebulae (Stothers 1963*a;* Chiu and Chin 1965; Vila 1965; Chin *et al.* 1966*b;* Stothers 1968), (3) frequency of type II supernovae (Stothers 1963*c*), and (4) isotopic abundances of heavy elements (Stothers and Chiu 1962; Fowler and Hoyle 1964).

It therefore seems worthwhile to re-examine the apparent paradox posed by the red supergiants. In the present paper, an investigation is made of the distribution on the H-R diagram of supergiants in all well-observed clusters and associations (§§ II–V). The theoretical model sequences are then discussed (§ VI), followed by assessment of the neglected features (§ VII). Comparison of observational and theoretical results concludes the paper (§§ VIII and IX).

II. RED SUPERGIANTS IN GALACTIC CLUSTERS AND ASSOCIATIONS

Galactic clusters and associations containing known M supergiants are listed in Table 1. All supergiant members (luminosity class I) are given in the table. Copious notes are appended at the end, and they include the chief source which lists the spectral types, V-magnitudes, and $B - V$ colors of the stars (the chief source is listed in parentheses after the cluster or association name). Distance moduli and mean color excesses for the groups were obtained from one or more of the following sources: the chief source; Johnson *et al.* (1961); Becker (1963); Buscombe (1963); Hoag and Applequist (1965); and the appended notes. The discrepancy among the various sources was quite small, except in the case of NGC 4755, for which the chief source was used. Usually a straight average of the various determinations was adopted, although some personal judgement was occasionally required.

The observed magnitudes of the O, B, and A supergiants have been corrected for interstellar extinction by using the intrinsic colors of supergiants as listed by Johnson (1966) and the customary ratio of total-to-selective extinction $A_V/E_{B-V} = 3.0$. Extinction corrections for the F–M supergiants have been obtained by using the mean color excess of the stellar group in which a given supergiant is found. An alternative procedure would be, of course, to use assumed intrinsic colors and the normal reddening law (e.g., Sharpless 1958). About half of the M supergiants are known variable stars with photographic amplitudes of typically 1 mag. Uncertainty still exists as to which phase is characteristic of the unperturbed star (Plaut 1965); we have adopted the median magnitude.

Bolometric corrections have been adopted from Johnson's (1966) tabulation. We have assumed B.C. $= -2.8$ for an O9.5 supergiant.

Group membership of the stars in Table 1 has been based on the following criteria: (1) position in equatorial coordinates with respect to the group center, (2) radial velocity, and (3) proper motion. Radial velocities (usually from Wilson 1953) are available for all stars except those noted under "Remarks." Proper motions (usually from Smithsonian Institution 1966) have been used as criteria for the stars in the following groups: h and χ Per, I Per, I Ori, I Gem, Cr 121, and II Sco.

Table 1 is believed to be essentially complete for all the supergiants in galactic clusters

TABLE 1

SUPERGIANTS IN GALACTIC CLUSTERS AND ASSOCIATIONS WITH RED-SUPERGIANT MEMBERS

Cluster	$(m-M)_0$	$\bar{E}_{B-V}$	Star	Sp.	M_v	M_{bol}	Remarks
NGC 457..	12.2	0.5	HD 7902	B6 Ib	−6 7	−7.5	
			ϕ Cas	F0 Ia	−8.7	−8 6	
			BD+57°258	M0 Ib–II	−5 1	−6 3	No radial velocity
NGC 581	12.1	0 4	HD 9311	B5 Ib	−6 0	−6 9	
			Zug 4	M2	−4 8	−6 2	No radial velocity
h Per.....	11.7	0.7	Oo 1162	B2 Ia	−7.1	−8 6	
			Oo 1057	B3 Ia	−7 0	−8 3	
χ Per.....	12 0	0 5	Oo 2227	B2 Ib	−5 5	−7 0	
			RS Per	M4.5 Iab	−5 2	−8 0	
I Per. . (inner)	12 0	0 5	Oo 16	B1 Iab	−7 0	−8 8	
			HD 13659	B1 Ib	−5.7	−7 5	No radial velocity
			Oo 662	B1 Ib	−5.4	−7 2	
			HD 14818	B2 Ia	−7.2	−8 7	
			HD 13841	B2 Ib	−5 8	−7 4	
			HD 13866	B2 Ib	−5 6	−7 2	
			Oo 2621	B8 Ia	−6 9	−7 5	
			HD 14899	B8 Ib	−6 0	−6 6	
			Oo 2178	A1 Ia	−7 3	−7 6	
			Oo 2589	A2 Ia	−6 6	−6 8	
			HD 14580	M0 Iab	−5 7	−6 9	
			BD+56°595	M0.5 Iab	−5 3	−6 5	
			FZ Per	M1 Iab	−5 2	−6.5	
			HD 14404	M1 Ib	−5 9	−7 2	
			AD Per	M2 5 Iab	−5 6	−7 2	
			HD 14826	M3 Iab	−5 8	−7 6	
			SU Per	M3 5 Iab	−5 4	−7 5	
			BU Per	M3 5 Ib	−5 1	−7.2	
I Per . (outer)	11.8	0.8	HD 13402	B0 5 I	−6 3	−8 3	
			HD 16310	B1 II:	−6 4	−8 2	No radial velocity
			HD 15690	B1 5 Ib	−6 4	−8 1	
			HD 14956	B2 Ia	−7 3	−8.8	
			HD 13267	B5 Ia	−6 7	−7 6	
			HD 15497	B6 Ia	−7 4	−8 3	
			HD 17145	B8 Ia	−6 2	−6 8	No radial velocity
			HD 15620	B8 Iab	−6 3	−6 9	No radial velocity
			HD 13744	A0 Iab	−6 4	−6 8	
			HD 12953	A1 Ia	−7 9	−8 2	
			HD 16778	A2 Ia	−6 6	−6 8	
			HD 13476	A3 Iab	−6 9	−7 0	
			HD 15316	A3 Iab	−6 7	−6 8	
			HD 17378	A5 Ia	−7.9	−7 9	
			HD 17971	F5 Ia	−6 4:	−6 3:	No radial velocity
			HD 14662	F7 Ib	−7 7:	−7 6:	
			HD 18391	G0 Ia	−6 7:	−6 7:	
			HD 11544	G2 Ib	−7 2:	−7 2:	No radial velocity
			HD 17306	K3 Iab+B:	−6.4:	−6 9:	No radial velocity
			BD+58°373	M0	−4 4	−5 6	No radial velocity
			BD+57°524	M0 5	−4 4	−5 6	No radial velocity
			BD+58°445	M1	−5 1	−6 4	No radial velocity
			K100172	M1	−2 8	−4 1	No radial velocity
			T Per	M2 Iab	−4 7	−6 1	
			BD+60°478	M2 Iab	−4.0	−5 4	No radial velocity
			HD 13136	M2 Ib	−5 2	−6 6	
			YZ Per	M2 5 Iab	−6 7	−8 3	
			S Per	M4 Ia	−4 5	−6 9	
I Ori......	8.2	0.1	ζ Ori	O9 5 Ib	−6 7	−9 5	
			ϵ Ori	B0 Ia	−6 7	−9 0	
			κ Ori	B0 5 Ia	−6 3	−8 3	
			β Ori	B8 Ia	−8 0	−8.6	
			α Ori	M2 Iab	>−7 8	......	Foreground member

TABLE 1—*Continued*

Cluster	$(m-M)_0$	$\bar{E}_{B-V}$	Star	Sp	M_v	M_{bol}	Remarks
I Gem.....	10 5	0.5	HD 41117	B2 Ia	−7.2	−8 7	
			HD 42087	B2 5 Ib	−5 8	−7 2	
			HD 43384	B3 Iab	−6 0	−7 3	
			HDE 250290	B3 Ib	−5 4	−6 7	
			BU Gem	M1 Ia	−5 2	−6 5	
			TV Gem	M1 Iab	−5 1	−6 4	Possible member
			WY Gem	M2 Iab+B	−4 7	−6 1	
Cr 121. ..	9 0	0 03	o^2 CMa	B3 Ia	−6 1	−7 4	
			η CMa	B5 Ia	−6 7	−7 6	
			δ CMa	F8 Ia	−7 3	−7.2	
			o^1 CMa	K3 Iab	<−5 3	<−5 8	Member?
			σ CMa	M0 Iab	−5 7	−6 9	Probable member
NGC 3293 .	12 1	0 3	HD 91969	B0 Ib	−6 4	−8 7	
			HD 91943	B0 5 Ib	−6 3	−8 3	
			CPD−57°3502	M0 Iab	−5 7	−6 9	
NGC 3766 .	11.4	0.2	HD 100943	B5 Ia	−4 9	−5 8	No radial velocity
			CD−60°3621	cM0	−4 8	−6 0	No radial velocity
			CD−60°3636	cM0	−4 6	−5 8	No radial velocity
IC 2944....	11.5	0 3	HD 101333	B0 Iab	−3 6	−5 9	
			HD 101545f	B0 5 Ib:	−4 7:	−6 7:	
			HD 101332	B1 Ib	−4.9	−6 7	
			HD 101712	M2 Ib+B?	−4 6	−6 0	
NGC 4755.	11.9	0.4	HD 111934	B1.5 Ib	−6 0	−7.7	
			HD 111990	B2 Ib	−6.3	−7 8	
			HD 111973	B3 Ia	−7 0	−8 3	
			HD 111904	B9 Iab	−7.1	−7 6	
			HD 111613	A1 Ia	−7.3	−7 6	
			CPD−59°4459	M2 Ib	−5 5	−6 9	
II Sco......	6 1	0.2	σ Sco	B1 III	−4 4	−6 0	
			α Sco	M1 Ib+B	−5 4	−6.7	

NOTES TO TABLE 1

NGC 457 (Pesch 1959).

NGC 581 (Hoag and Applequist 1965): The M star is listed in Zug's (1933) catalogue. Its magnitude is taken from Hoag *et al.* (1961). Although the star HD 9365 (F4) lies within the cluster area, its faintness and low radial velocity (Wilson 1953) probably rule it out as a cluster member.

I Per, h and χ (Wildey 1964): The double cluster and its surrounding association have recently been discussed by Schild (1967), who distinguished in more detail than previously an "inner group" surrounding the nucleus of χ Per and a broader "outer group" extending toward the nucleus of h Per. We have listed those supergiants identified by Schild as being members of the various groups. (Schild apparently omits six early-type supergiants from Wildey's list for the general region but includes five intermediate-type supergiants which are located on the periphery of the region.) The list of supergiants is essentially complete (Wildey 1966). The mean reddening for each group was obtained by using Wildey's observed colors of the B supergiants and the intrinsic colors listed by Johnson (1966). We have adopted Bidelman's (1957) spectral types and HD magnitudes for the F–K supergiants; for their extinction, we have used the mean reddening based on the B supergiants (although seven O stars in the outer group yield $\bar{E}_{B-V} = 0.7$ on the assumption that $(B - V)_0 = -0.32$). The extinction for the M supergiants was obtained from the reddening of faint neighbors as measured by Wildey. Schild has suggested that the M supergiants and many of the B–A supergiants in the outer group may be background stars in the Perseus arm with $(m - M)_0 = 12.7$, but that possibility can be neither proved nor disproved. The pulsational characteristics of YZ Per seem reasonable with the adopted (or a slightly smaller) distance modulus of I Per, while the anomalous characteristics of T Per and S Per would be removed if these stars were at a greater distance (Stothers 1969). Radial-velocity criteria in the case of some of the M supergiants have been derived from Bidelman's (1947) remark concerning the results of his unpublished measurements. Proper motions of many of the stars have been studied by Oosterhoff (1937) and by Meurers and Aksoy (1960) and have been rediscussed recently by Schild.

I Ori (Sharpless 1952): Magnitudes and colors are from Iriarte *et al.* (1965). The positions of β Ori and α Ori are not central, but the nebulosity associated with them (Morgan, Strömgren, and Johnson 1955) and their radial velocities (Wilson 1953) suggest membership in I Ori. However, the proper motion of

NOTES TO TABLE 1—*Continued*

α Ori (Smithsonian Institution 1966) is discrepantly large, and unless it is a runaway star (moving NE. out of the association at 65 km sec^{-1}), α Ori should not be considered a definite member. In fact, its trigonometric parallax (Jenkins 1963), although weakly determined, yields an absolute magnitude of only $M_v = -6.1$, and the intensity of the interstellar K-line in its spectrum (Wilson 1959) yields $M_v = -5.7$ (as calibrated from spectroscopically similar M supergiants in I Per). Furthermore, its pulsational characteristics also suggest $M_v \sim -6$ (Stothers 1969). Blanco (1963) has searched the region of I Ori and found no other M supergiant members. In forming $(m - M)_0$ for the association, Sharpless' (1962) recent determination was taken into account.

I Gem (Hardie, Seyfert, and Gulledge 1960): The grouping of blue stars called I Gem may not be a single association since the luminosity class V stars are distributed over a wide range of magnitude and area. However, the class I and II stars show a distinct clustering in area. Hardie *et al.* (1960) list eleven supergiants, including Hiltner's (1956) HDE 248587, and five bright giants in the spectral range B0–A0. We may form distance moduli for these stars by subtracting Blaauw's (1963) absolute magnitudes (which are given as a function of spectral type and luminosity class) from the extinction-corrected observed magnitudes. Two distinct groups emerge, except for two near and two distinct stars. Three of the discrepant stars are located farthest away from the group center in equatorial coordinates. Proper motions (Smithsonian Institution 1966) are available for all of the stars, and radial velocities for eleven stars (Wilson 1953; Petrie and Pearce 1962). Five stars (including the three farthest) may be ruled out as group members on the basis of their space motions. The ten stars considered as members are HD 40003, 40297, 40589, 41117, 42087, 42400, 43384, 43818, 43837, and 250290. Median values of $V_0 - M_v$, with their maximum deviations, are 10 5 $\pm$ 0.1 (two stars of class II), 10.5 $\pm$ 0.3 (four stars of class I), and 11.7 $\pm$ 0.1 (four stars of class I). The color excesses show much scatter but average about 0.5 for the near group. Apparently, the bright giants belong to the near group of supergiants, for which we shall adopt $(m - M)_0 = 10\ 5$. It is interesting that five of these six stars in the near group are the same stars which were originally ascribed to I Gem by Crawford *et al.* (1955). For the three M variables, median visual or photographic magnitudes and spectral classifications have been taken from the GCVS (Kukarkin *et al.* 1958); we have assumed that C.I. = +2.2. Two of the M variables lie in the near group according to their apparent brightness and proper motion. Although the proper motion of TV Gem is discrepant with both groups, its pulsational characteristics are normal if it is an association member (Stothers 1969).

Cr 121 (Feinstein 1967): The proper motion of o^1 CMa and the radial velocity of σ CMa, are slightly discrepant. The reddening of o^1 CMa, as determined from the intrinsic color listed for a K3 supergiant by Johnson (1966), is 0 38 mag, and the star may not be a cluster member despite its central location, compatible radial velocity, and apparently compatible $M_v = -5\ 2$ based on the K-line (Wilson and Bappu 1957). The peripheral star HD 56618 (gM3) agrees in radial velocity but not in proper motion with the mean of cluster members.

NGC 3293 (Feast 1958).

NGC 3766 (Sher 1965): The spectral type of HD 100943 is from Morgan, Code, and Whitford (1955). Three other luminous OB stars in NGC 3766 are probably bright giants. The star CD$-60°$3630 (cM1) seems too distant to be a cluster member (Blanco and Münch 1955).

IC 2944 (Thackeray and Wesselink 1965): The supergiants are all in the H II region surrounding the O cluster IC 2944. The mean cluster reddening was assumed for two of the B supergiants: HD 101545f (IDS magnitude) and HD 101332. The supergiants seem to belong to an older population than the O stars.

NGC 4755 (Feast 1963): The mean cluster reddening was assumed for the A supergiant HD 111613. Feast's distance modulus has been adopted.

II Sco (Hardie and Crawford 1961): The magnitude and color of the β Cephei star σ Sco are from Iriarte *et al.* (1965). This is the most luminous B star in II Sco. The radial velocity (Wilson 1953) and proper motion (Smithsonian Institution 1966) of the M variable α Sco suggest that it is an association member. Its median visual magnitude and spectral type are from the GCVS (Kukarkin *et al.* 1958).

or associations which contain known M supergiants. Investigation of additional stellar groups will undoubtedly increase the number containing M supergiants. For example, Blanco *et al.* (1955) found four early M stars, probably supergiants, near the distant cluster NGC 7419; no other spectroscopic or photometric data are available for this cluster. The M supergiant ψ^1 Aur lies in the vicinity of a group of four OB stars and seems to share their common motion (Blanco 1954). The composite M supergiant HD 188037 may lie in the I Vul association (Deutsch 1960). Ambartsumian (1953) has tentatively placed seven other M supergiants in the Cassiopeia, Cepheus, and Cygnus associations. In a study of late-type stars along the galactic equator between Puppis and Crux, Blanco and Münch (1955) have found several definite groupings of M supergiants; among them are: two cM stars near I Pup, three cM stars a few degrees from I Vel, and five cM stars near the η Car nebulosity. In another investigation based primarily on the surveys made by Blanco, Nassau, and Morgan, Sharpless (1965) found additional groupings of M supergiants along most of the remaining section of the galactic equator. Some of them may belong to O associations (Sharpless 1966). An attempt at obtaining distance resolution of the groupings has been made by Westerlund (1964). The lack of identification of many groupings with known associations is probably due to the dissolution of old associations (which contain M supergiants) into the general field.

The conclusion that M associations are old rests on: (1) their relatively great spatial extent; (2) their poorness in gas and dust, based on the observed absence of H II regions (Westerlund 1960; Sharpless 1965) and on their small average reddening $\langle E_{B-V} \rangle = 0.3$ (Table 1); and (3) not independently, the absence of stars earlier than O9, except in the case of IC 2944 and I Per (cf. also Bidelman 1954; Sharpless 1965). A search of six of the youngest O clusters and associations did not turn up any previously unidentified M supergiants (Blanco and Grant 1959; Blanco 1963). In fact, some of the clusters are so young that they do not even contain blue supergiants.

The carbon stars of class N and the spiral-arm subgroup of S stars show a strong concentration toward the galactic plane and an affinity for obscured regions (Blanco 1965). It is known that the N stars have a clustering tendency (Smith and Smith 1956; Ishida 1960), although there is a suggestion that they avoid OB associations (Westerlund 1964). However, the variable star RS Cyg (N0ep) may be an escaped member of the O cluster IC 4996 (see discussion in Reddish 1967), and another N star may belong to NGC 7419 (Blanco *et al.* 1955; Sandage [quoted in Nassau 1958]). The spiral-arm S stars also have a clustering tendency (Smith and Smith 1956), which agrees well with the surface distribution of OB associations (Westerlund 1964). Four S stars are known to lie in the η Car nebulosity (Blanco and Münch 1955) and two others lie near the O cluster NGC 6530 (Pik Sin The, quoted in Blanco 1965). Since the N and S stars are very rare, it is not surprising that their absolute magnitudes are poorly known. Keenan (1942) has suggested that the S stars form a later extension of the brightest M supergiants, which seem to cut off at M4.5 (cf. also Deutsch 1956*a*).

The data on the brightest supergiants of intermediate spectral type F, G, or K are sparse, at least regarding cluster membership. Such objects are rare and probably mostly absent from young clusters (but see Table 1). The Cepheids in the region of I Per, if they are members, belong to an older group composed of stars of lower luminosity (Bidelman 1943; Eggen 1965). Several F and G supergiants are found in the vicinity of the η Car group (Bidelman 1954), which is dominated by the F supergiant η Car itself (Feinstein 1963). In general, yellow supergiants (including Cepheids) tend to inhabit clusters of intermediate age—an observation which is supplemented by the statistics of field supergiants in the Galaxy (Bidelman 1958; Kraft and Hiltner 1961) and in the Magellanic Clouds (below).

III. RED SUPERGIANTS IN THE MAGELLANIC CLOUDS

The Magellanic Clouds contain many bright star groups which are designated as clusters but are probably small associations. *UBV* photometry is now available for many of these groups, as well as for the giant "constellations." Spectroscopic data are usually lacking, except for several surveys of the brightest M stars, which cover most of the Large Cloud area. The stellar distribution shows a clustering tendency of blue and red supergiants, with a scattering of yellow stars. Many of the brighter yellow stars are probably foreground objects in our own Galaxy, on the basis of star counts, proper motions, radial velocities, and luminosity classifications (see summaries in Kerr and Rodgers 1964). The scanty spectroscopic evidence suggests that most of the remaining bright yellow stars may be unresolved blends of B and M supergiants (Feast 1964). Hence the Cloud groups resemble those in our own Galaxy.

The Small Cloud cluster NGC 330 (Arp 1959) has nearly the same number of blue (ten) and red (twelve) supergiants as the inner group of I Per. However, the blue supergiants in NGC 330 lie in a more limited spectral range B5–A1 (Feast 1964) than those in I Per. Unfortunately, spectra are not available for the red supergiants. Near the 30 Dor nebulosity in the Large Cloud, Shapley and Nail (1948) found fifteen variable "red" supergiants. These are comparable in number, variability, and absolute magnitude with the M supergiants in I Per (Sharpless 1958).

The only spectroscopic information on red supergiants in Cloud associations has been tabulated by Westerlund (1961) for several associations in the Large Cloud. We have listed in Table 2 those stars, with one exception, for which decimal subdivisions of the

TABLE 2

M SUPERGIANTS IN CLUSTERS AND ASSOCIATIONS OF THE LARGE MAGELLANIC CLOUD (WESTERLUND)

Cluster	$\bar{A}_v$	Star	Sp	M_v	M_{bol}	Remarks
NGC 1962–5–6–70	0 4	W29	M	−7 2	−8 5:	No radial velocity
NGC 1994 .. .	.6	W7	M1	−6 5	−7.8	No radial velocity
NGC 2004.... .	.0	W1	M0 5	−6 0	−7 2	
NGC 2011 ..	.0	W24	M2	−5 1	−6 5	No radial velocity
NGC 2021.. .	0	W58	M0:	−5 7	−6 9	No radial velocity
		W75	M1	−5.8	−7.1	No radial velocity
		W22	M2	−6 5	−7 9	No radial velocity
NGC 2100 .	45	W5	M4	−4.7	−7 1	No radial velocity
Anon b2 ...	.45	W1	K–M0:	−5.2	−6 4	No radial velocity
Anon b4 ...	0 25	W12	M0+:	−5.5	−6 7	No radial velocity
		W39	M0 5	−5 7	−6 9	No radial velocity
		W80	M2	−6.2	−7.6	No radial velocity

NOTES TO TABLE 2

The list of red supergiants for each cluster or association is very incomplete, except where noted below.

NGC 1962–5–6–70: The star W29 is the only red supergiant in the association. A blue star, W50 (R103 or HDE 269546, spectral type B3 Ip), is the most luminous blue supergiant in the Magellanic Clouds. (Westerlund [1961], however, lists its spectral type as Of.) Its radial velocity is listed by Feast *et al.* (1960).

NGC 2004: The cluster extinction has been taken from Woolley (1960). The star W1 (R108) has an anomalously blue $B - V$ color and is classified as "peculiar" by Feast *et al.* (1960); it is probably blended with an early-type supergiant. The radial velocities of W1 (R108) and of a blue supergiant W20 (R109, spectral type B6 I) are listed by Feast *et al.* (1960).

Anon b2: The cluster extinction is assumed to be equal to the mean extinction of all the clusters in Westerlund's (1961) region "b." The star W1 is the only red supergiant in the cluster.

Anon b4: The three listed stars are the only red supergiants in the cluster.

M spectral type (Case system) are available (Westerlund 1960). Spectral types of a few of the blue stars are also available (Feast, Thackeray, and Wesselink 1960). We have adopted the Westerlund or Radcliffe running star number and Westerlund's values of the mean cluster extinction. The true distance modulus to the Large Cloud has been taken to be $(m - M)_0 = 18.7$ (Stothers and Simon 1968).

Color-magnitude diagrams of still other Cloud associations have been prepared by Hodge, Bok, Woolley, and their collaborators (see references in Thackeray 1963; Bok 1966). The bolometric magnitudes of the brightest stars can be crudely estimated, and, in general, they are comparable for the red and blue supergiants in a given association. The group NGC 1962–5–6–70 in the Large Cloud (Westerlund 1961) contains the brightest red star listed in a Cloud association. The star, W29, is known to be of spectral type M and is the only red star in the area of the association, which contains three very bright blue stars. If members, two of the blue stars have masses of $\sim 25\ \mathfrak{M}_\odot$. However, the third blue star, which is known spectroscopically to be a member of the Large Cloud, has a mass of $\sim$60 $\mathfrak{M}_{\odot}$ and is the most massive blue supergiant in either Magellanic Cloud (Stothers and Simon 1968). The youngest clusters in Westerlund's study, NGC 1983 and NGC 2014, do not contain any supergiants at all.

Still brighter M stars are known in the field of the Large Cloud. Westerlund (1964) has plotted a map showing the distribution of red stars over the major portion of the Large Cloud; his survey is nearly complete for M0–M4 stars down to infrared magnitude $I = 14$. Most of the early M stars are brighter than $I = 12$ and some are variable. The only tabular material available covers four small regions, totaling 11.2 square degrees, including the 30 Dor region (Westerlund 1960). Table 3 has been prepared from Westerlund's lists. It is apparent that the brightest M stars in the Large Cloud have early spectral types (M0–M4), just as in the case of our own Galaxy. This observation is reinforced by the bright group of stars with $8 < I < 10$, which have M0–M2 spectral types almost exclusively. However, it is not true of the stars brighter than $I = 8$, of which half have spectral types in the range M4–M8. Westerlund (1964) suspects that the stars with $I < 9$ are foreground galactic objects.

TABLE 3

INFRARED LUMINOSITY FUNCTION OF M SUPERGIANTS IN 11.2 SQUARE DEGREES OF THE LARGE MAGELLANIC CLOUD (WESTERLUND)

I	M0–M4	M5–M8	M	Total
5.00–5 99 ..	0	1	0	1
6 00–6 99 .	2	0	1	3
7 00–7 99 .	3	0	1	4
8 00–8 99 .	5	0	1	6
9 00–9.99.	10	1	3	14
10 00–10 99	70	0	11	81
11 00–11 99	54	3	29	86
12 00–12 99	11	28	21	60
13 00–13 99	0	17	21	38
Total..	155	50	88	293

Using comparison fields, he estimates that one or two M stars per square degree belong in our Galaxy. Radcliffe spectra taken of some of his brightest stars have, in fact, not uncovered any true Cloud members (Thackeray 1963). It is therefore significant that in Table 3 the number of stars shows a sharp decrease brighter than $I = 10$. The precise upper limit of brightness for true M members of the Large Cloud is, however, impossible to determine with the presently available material.

The brightest stars of intermediate spectral type in the Clouds are four F and G red "super-supergiants" belonging to the Large Cloud (Feast *et al.* 1960). Additional members of this class have been found (e.g., Fehrenbach, Duflot, and Duflot 1965) but they are not as bright as these four. The masses of the four are all about 30 $\mathfrak{M}_{\odot}$ (Stothers and Simon 1968). Thirty-four supergiants in the spectral range B0–A5 (twenty-three in the Large Cloud) are known with masses in excess of 30 $\mathfrak{M}_{\odot}$ (Feast *et al.* 1960; Stothers and Simon 1968). Further spectroscopic data are expected to increase this number somewhat (Thackeray 1962; Fehrenbach *et al.* 1965).

IV. OBSERVATIONAL ERRORS

The values of M_{bol} listed in Tables 1 and 2 are subject to various sources of error. These sources are given in Table 4. We have not explicitly included the error due to our adoption of the fixed ratio $A_V/E_{B-V} = 3.0$.

The greatest source of error lies in the uncertainty associated with the bolometric corrections. The bolometric corrections of the blue supergiants may be in error by typically half a magnitude if one simply compares the theoretical values listed by Harris (1963) with the less negative observational values listed by Johnson (1966). For the red supergiants, the error is probably also half a magnitude, as seen by comparing Smak's (1966) bolometric corrections with Johnson's (1966) in the case of M giants. Hence the earliest B and latest M supergiants will have the most uncertain luminosities. The errors average ± 0.6 and ± 0.7 mag in the case of the B and M supergiants, respectively. If the adopted bolometric corrections happen to be nearly correct and if the M variables have "normal" luminosities at median light, then the errors are only ± 0.3 mag for both B and M supergiants.

TABLE 4

ESTIMATED ERROR OF ABSOLUTE BOLOMETRIC MAGNITUDES

Source of Error	Blue (mag)	Red (mag)
Observed magnitude	±0 05	±0 10
Light variability....	......	± .50
Visual extinction		
Adoption of mean cluster reddening	...	± .20
Spectral subdivision....	± 05	....
True color.	± .10	....
Bolometric correction	± 50	± .50
Cluster distance modulus.	± .25	± 25
Combined error. . .	±0 57	±0.71

V. MASSES

Masses for the blue supergiants have been estimated from their absolute bolometric magnitudes by using the theoretical mass-luminosity relation for massive core helium-burning stars (Iben 1966*a;* Stothers 1966*a;* and Stothers and Chin 1968). The initial chemical composition which has been adopted in the theoretical model calculations is $X_e = 0.70$–0.71 (hydrogen) and $Z_e = 0.02$–0.03 (metals). The hydrogen abundance is close to the value $X_e = 0.67$–0.70 estimated for extreme Population I objects by Percy and Demarque (1967) using a variety of observational and theoretical evidence. Although massive stars evolve at nearly constant luminosity in the blue region during core helium burning, some uncertainty in the luminosity for a given mass exists because of (1) a slight evolutionary brightening, (2) inaccuracies in the adopted opacities, and (3) uncertainty about the degree of chemical homogeneity in the envelope. However, the effect of these uncertainties is rather small. The calculated luminosities for the various masses (and their uncertainties) are shown in the accompanying tabulation.

$\mathfrak{M}/\mathfrak{M}_\odot$	M_{bol}
9 ...	−5 65±0 10
15	−7 40±0 10
30 ..	−9 10±0 05
60....	−10 45±0 02

It should be explicitly remarked that, for a given initial chemical composition, there exists a one-to-one correspondence between the luminosity and mass of a blue supergiant.

The probable error of the observed absolute bolometric magnitudes, ± 0.7 mag, results in a non-uniform probable error of the estimated masses. The errors in the masses are ± 2, ± 5, ± 15, and ± 25 $\mathfrak{M}_\odot$, for masses of 10, 20, 40, and 60 $\mathfrak{M}_\odot$, respectively. A less stringent estimate of the magnitude error, ± 0.3 mag, results in probable errors in the masses of ± 1, ± 2, ± 6, and $\pm 10\ \mathfrak{M}_\odot$, respectively.

Masses for the supergiants in the Large Magellanic Cloud can be estimated in the same way as for the galactic supergiants. The effect of the slightly different Cloud chemical composition, $X_e = 0.75$ and $Z_e = 0.02$–0.03 (Stothers and Simon 1968), would be to increase the mass at a given luminosity in the following way, namely, to add 0.25, 1, 2, and 4 $\mathfrak{M}_\odot$ to 10, 20, 40, and 60 $\mathfrak{M}_\odot$, respectively.

Masses for the red supergiants cannot be directly obtained from their absolute bolometric magnitudes. The reasons for the difficulty here are (1) ambiguity regarding the evolutionary stage of the red supergiants, viz., core helium burning or core carbon burning (or even later stages), (2) evolutionary brightening and fading of red-supergiant models, (3) overlap in luminosity of high and low masses (Stothers and Chin 1969). Consequently, we shall identify the masses of red supergiants with the masses of blue supergiants in the same cluster or association, whenever this is possible. Such an identification is reasonable since the difference in lifetimes at advanced stages of evolution ought to be short compared with the cluster age.

VI. THEORETICAL EVOLUTION OF BLUE AND RED SUPERGIANTS

The presence of red supergiants in young clusters and associations has aroused much speculation about the stage of their evolution. Explanations have generally fallen into five categories of possible stages: (1) pre-main-sequence contraction, (2) core helium burning, (3) internuclear core contraction, (4) core carbon burning, and (5) core oxygen burning. Probably the only suggestion that has not been made is that they are in the core hydrogen-burning stage! We shall discuss each possibility in turn.

a) Pre-Main-Sequence Contraction

Reviving the old giant-dwarf theory, Shapley (1955) suggested years ago that the bright red supergiants in the great "constellations" of the Large Magellanic Cloud may be in a stage of gravitational contraction toward the main sequence. Iben (1967) recently made the same suggestion in the case of the h and χ Per association. Noting the wide dispersion in evolving masses off the main sequence in this association, Iben supposed that a similarly wide dispersion of contracting masses might provide the explanation for the large number of red supergiants. Nevertheless, the red supergiants comprise a rather compact group at high luminosity, compared with the scatter of lower-luminosity (definitely contracting) stars distributed across the Hertzsprung gap (Wildey 1964). If contracting, the red supergiants would have an age of only hundreds of years (Iben 1965; Ezer and Cameron 1967) and far from comparable (for the observed numbers of stars) with the age of the brightest blue stars, namely, millions of years. Evans (1968) has pointed out other spectroscopic and photometric differences as compared with known contracting stars. If the group were unique among young clusters, the newly-formed-group hypothesis would be a possibility. Since many young clusters, however, show a group of red supergiants, the hypothesis is practically untenable.

b) Helium Burning

The available model sequences for massive stars that include the helium-burning phase of evolution are: 9 $\mathfrak{M}_\odot$ (Iben 1964, 1966*a;* Hofmeister 1967), 15 $\mathfrak{M}_\odot$ (Iben 1964, 1966*b*), 15.6 $\mathfrak{M}_\odot$ (Sakashita, Ono, and Hayashi 1959; Hayashi, Jugaku, and Nishida 1959*a*, *b*, 1960; Hayashi and Cameron 1962*a*, *b*, 1964; Hayashi *et al.* 1962; Hayashi 1966), and 15, 30, 60, and 100 $\mathfrak{M}_\odot$ (Stothers 1963*b*, 1964, 1965, 1966*a*, *b;* Stothers and Chin 1968). In all cases, rotation and mass loss were neglected. The mass, physical assumptions, and helium-burning lifetime for each sequence are listed in Table 5. The basic stellar structure comprises a dense helium core and an extended hydrogen-rich envelope, at the bottom of which hydrogen burns in a thin shell.

The location on the H-R diagram of stellar models with extended envelopes and nuclear-burning shells is very sensitive to relatively small changes in basic physical input parameters. These parameters include: nuclear reaction rates, initial chemical composition, opacity, internal mixing, and mass loss.

i) *Nuclear Reaction Rates*

Burbidge (1962) first mentioned the uncertainty in the helium-burning reaction rates which were used in calculating the original models of Hayashi and Cameron. Iben (1966*b*) investigated the effect of a large change in the $C^{12}+He^4$ rate and Stothers and Chin (1968) studied similar changes in the triple-alpha rate and in the CNO abundance of the hydrogen-burning shell. Only little observable change was found on the H-R diagram.

TABLE 5

THEORETICAL EVOLUTIONARY SEQUENCES OF MODELS FOR MASSIVE STARS DURING CORE HELIUM BURNING

$\mathfrak{M}/\mathfrak{M}_\odot$	X_e	Z_e	Opacity	Composition in Intermediate Zone	$\tau(10^6$ yr)	τ_{red}/τ	Author
9. . . .	0 60	0 04	$\kappa_a+\kappa_s$	Inhomogeneous	2 7	0 5	Hofmeister (1967)
9. . .	.74	.02	$\kappa_a+\kappa_s$	Inhomogeneous	4 1	1 0	Hofmeister (1967)
9.	.71	.02	$\kappa_a+\kappa_s$	Inhomogeneous	4 0	0.1	Iben (1966*a*)
15. . .	.71	.02	$\kappa_a+\kappa_s$	Homogeneous	1 6	0 0	Iben (1966*b*)
15. . . .	.70	.03	$\kappa_a+\kappa_s$	Inhomogeneous	1 1	0 2	Stothers and Chin (1968)
15.6. . .	.90	.02	κ_s only	Inhomogeneous	1.2	0.0	Hayashi and Cameron (1962*a*)
30. . . .	.70	.03	κ_s only	Inhomogeneous	0.53	0.0	Stothers (1966*a*)
60. . . .	0 70	0 03	κ_s only	Inhomogeneous	0.34	0.0	Stothers and Chin (1968)

ii) *Initial Chemical Composition*

Burbidge (1962) and Iben (1967) recommended changes in the initial hydrogen and metals content to shift the star into the red-supergiant region. However, Stothers and Chin (1968) showed that the resulting changes in the mean molecular weight and opacity could not *alone* make the blue models red.

iii) *Opacity and Internal Mixing*

Apparently, it is the *opacity source and chemical inhomogeneity together* which are crucial for determining the stellar radius. The chemical inhomogeneity arises from the depletion and redistribution of hydrogen that take place in the convective core and in the convectively unstable zone just outside the core during hydrogen burning. The consequences for core helium-burning models with hydrogen-burning shells are as follows. Models in which the opacity source is taken to be solely electron scattering and/or in which the unstable intermediate zone is treated as fully convective start helium burning as blue supergiants. With the inclusion of bound-free absorption and semiconvective mixing, the models start as red supergiants. The latter alternative seems to be the correct one from a theoretical viewpoint (Sakashita *et al.* 1959; Kato 1967; Stothers and Chin 1968).

The assumption of semiconvective mixing results in a chemical inhomogeneity quite similar to the inhomogeneity which is obtained on the assumption of no mixing at all. Therefore, models calculated on the no-mixing assumption but including bound-free absorption should also start helium burning as red supergiants. Model sequences covering evolution up to the onset phase of helium burning have been calculated in several cases with the no-mixing assumption: 15 $\mathfrak{M}_\odot$ (Hartwick 1967), 16 $\mathfrak{M}_\odot$ (Paczynski 1967), and 30 $\mathfrak{M}_\odot$ (Kotok 1966). In all cases, the last calculated models are in a stage rapidly approaching the red-supergiant region. The analogous models calculated with the semiconvective assumption are 15, 60, and 100 $\mathfrak{M}_\odot$ (Stothers and Chin 1968). Thus, the results for massive stars are similar to those for stars of slightly lower mass (9 $\mathfrak{M}_\odot$) which have radiative inhomogeneous zones and also begin helium burning as red supergiants (Iben 1966*a;* Hofmeister 1967).

Available calculations show that, for all masses and most of the assumed initial chemical compositions, the models subsequently evolve into the region of blue or yellow supergiants. However, according to results of Hofmeister (1967) for models of 9 $\mathfrak{M}_\odot$, a relatively small change in initial chemical composition can cause drastic changes in the ratio of times spent in the blue and red regions (presumably through the effect of opacity). For one of Hofmeister's initial compositions, the star never leaves the red-supergiant region! However, Iben's (1966*a*) models of 9 $\mathfrak{M}_\odot$, with nearly the same initial composition as Hofmeister's, *rapidly* leave the red-supergiant region; we must presume that opacity-formula and mixing-length differences are the reason for the discrepancy. Nevertheless, while such an extreme result as Hofmeister's is observationally ruled out for massive stars by the mere presence of blue supergiants, it does suggest a possibility of accounting for any large observed ratio of the number of red and blue supergiants.

iv) *Mass Loss*

The ejection of matter is a still more uncertain factor (Burbidge 1962). If the amount of mass lost is not too extensive, it tends to maintain the redness of a red supergiant and has little other major effect (Hayashi *et al.* 1962). Since the observed rates of mass loss (§ VII) are rather modest compared with the relevant evolutionary lifetimes, we may infer that the chief effect would be merely to increase the ratio of red to blue lifetimes.

In summary, we may account for at least some of the red supergiants as being stars in the phases of helium core contraction and early core helium burning. Although the fraction of the helium-burning lifetime spent in the red region is uncertain, the occurrence of such stars in this region is assured by the adoption of semiconvective mixing in the intermediate zone, by a reasonable initial metals abundance, and by the possibility of some mass loss.

c) Internuclear Core Contraction

The freezing of a stellar core by neutrino emission during the internuclear core-contraction phase following helium burning might substantially increase the lifetime of a red supergiant. Originally proposed (in a different context) by Chiu, Morrison, and Reeves (1962) and elaborated on by Stothers (1963*a*), the idea was supported by Iben (1966*b*) and Ruderman (1965), who envisaged helium shell burning as stabilizing the luminosity. Although cooling does occur in the 0.8 $\mathfrak{M}_\odot$ core of a 5 $\mathfrak{M}_\odot$ star (Weigert 1966), the 2.5 $\mathfrak{M}_\odot$ core in the Hayashi and Cameron star of 15.6 $\mathfrak{M}_\odot$ well exceeds the Chandrasekhar and Schönberg-Chandrasekhar limits and has since been shown definitely to contract into carbon burning, with or without neutrino emission (Hayashi *et al.* 1962; Hayashi and Cameron 1962*b*, 1964; Hayashi 1966; Murai *et al.* 1968). Carbon burning ought to occur directly in all red supergiants with core mass greater than $\sim$1 $\mathfrak{M}_\odot$ (Takarada, Sato, and Hayashi 1966; Murai *et al.* 1968).

d) Carbon Burning and Later Phases

Whether helium burning results in a core of predominantly carbon or oxygen depends on the reduced alpha-particle width of the 7.12-MeV level in O^{16}. The latest work suggests $\theta_\alpha^2 = 0.085 \pm 0.04$ for this level (Stephenson 1966; Loebenstein *et al.* 1967; Fowler, Caughlan, and Zimmerman 1967). Using $\theta_\alpha^2 = 0.10$, Stothers and Chin (1968) derived a final core carbon abundance of $X_C = 0.3$ for stars in the mass range 15–60 $\mathfrak{M}_\odot$. In this instance, massive stars experience a subsequent $C^{12} + C^{12}$ phase. Earlier considerations that helium burning may result in a core of virtually pure oxygen in red supergiants (Fowler quoted in Ruderman 1965; Chiu 1966) are probably no longer relevant. Observations of a significant C/O ratio in the atmospheres of N stars may be further evidence of a large carbon production.

The carbon-burning phase has been calculated in the case of 15.6 $\mathfrak{M}_\odot$ with and without the inclusion of neutrino emission. The star burns carbon as a red supergiant (Hayashi and Cameron 1962*a*, *b*; Hayashi *et al.* 1962; Cameron 1965; Hayashi 1966; Sugimoto *et al.* 1968). More detailed, but analogous, calculations have been made by Stothers and Chin (1969) for 15, 30, and 60 $\mathfrak{M}_\odot$. The inclusion of neutrino emission shortens the lifetimes by more than an order of magnitude, as was already predictable from the idealized models of Reeves (1963).

Stephenson (1966) and Fowler *et al.* (1967) have suggested that the rate for the $C^{12} + C^{12}$ reaction is still very uncertain and that carbon burning may actually take place at a lower temperature where neutrino emission is unimportant. However, the rate is extremely sensitive to temperature and would have to be in error by a factor of order 100 to make neutrino emission unimportant. At the relevant stellar temperatures, the error in the derived rate may be only a factor of 10 (Reeves 1965). In fact, the most

TABLE 6

THEORETICAL EVOLUTIONARY TIMETABLE FOR MASSIVE STARS (IN UNITS OF 10^5 YEARS)

COLOR	PHASE	15 $\mathfrak{M}_\odot$		30 $\mathfrak{M}_\odot$		60 $\mathfrak{M}_\odot$	
		Without ν	With ν	Without ν	With ν	Without ν	With ν
Blue/red....	Core He burning	11 0	11 0	5 3	5.3	3 4	3 4
Red...... .	Contraction of He core	0.7	0.7	0.2	0 2	0 1	0.1
Red........	Contraction of C/O core	0.2	0.1	0 07	0 05	0.03	0.02
Red........	Core C, Ne, O burning*	4–6	0.02–∼0	3–4	0.01–∼0	3–3	0.01–∼0
	Maximum $\tau_{\text{blue}}/\tau_{\text{red}}$	2	∼15	1.5	∼20	1	∼25

* Range indicates ($X_C = 1.0$, $X_O = 0.0$) and ($X_C = 0.0$, $X_O = 1.0$), respectively, at the end of core-helium burning

recent work on the rate (Arnett and Truran 1969) yields a burning temperature slightly *higher* than that given by Reeves's rate.

Stothers and Chin (1969) have made a detailed investigation of the evolution of massive stars beyond core carbon burning. They find that (1) the stars evolve as red supergiants, (2) the main nuclear-burning phases up to silicon photodisintegration are neon and oxygen burning, and (3) a firm lower limit may be placed on the evolution time without neutrino emission. Their results for 15 $\mathfrak{M}_\odot$ are similar to the estimates made earlier by Hayashi *et al.* (1962) for nearly the same mass. The URCA neutrino process, involving the beta decay of heavy isotopes, does not depend on the electron-neutrino interaction. However, it becomes an important energy-loss mechanism only during the advanced stages of silicon-to-iron burning (Chiu 1961*a*, 1963; Hayashi *et al.* 1962; Dallaporta and Saggion 1968) and may be neglected here.

Table 6 contains a *lower limit* on the lifetime after carbon core contraction without the electron-neutrino processes by considering only $C^{12} + C^{12}$, $Ne^{20} + \gamma$, and $O^{16} + O^{16}$ burning and by ignoring the other nuclear-burning and internuclear core-contraction phases. The lifetime including the electron-neutrino processes is determined essentially by the $C^{12} + C^{12}$ phase since the subsequent phases last only a few years at most (Reeves 1963; Stothers 1963*a*; Stothers and Chin 1969).

We conclude this section by suggesting that the red supergiants in young clusters and associations are evolving in (1) the phase of helium core contraction and early core helium burning and (2) the phase of carbon core contraction and carbon-to-iron burning.

VII. STELLAR WIND, ROTATION, AND DUPLICITY

The available model sequences in Table 5 have ignored effects of stellar winds, rotation, and duplicity (binaries). It is necessary to assess the importance of each effect before comparing the theoretical tracks with the observational data.

a) Stellar Wind

The total mass lost in the form of stellar winds is almost certainly negligible for main-sequence stars (Underhill quoted in Stothers 1963*c;* Underhill 1966; Williams 1967; Stothers and Chin 1968) as well as for blue supergiants (Morton 1967; Lucy and Solomon 1967; Stothers and Simon 1968). The only likely site for extensive mass loss is the red-supergiant region. Weymann obtained an observational estimate of $4 \times 10^{-6}\, \mathfrak{M}_\odot$ year^{-1} for α Ori ($\sim 25\, \mathfrak{M}_\odot$) and $6 \times 10^{-7}\, \mathfrak{M}_\odot$ year^{-1} for α^1 Her ($\sim 2\, \mathfrak{M}_\odot$) (Weymann 1962; Woolf 1963). Deutsch (1956*b*) and Wilson (1960) had earlier found rates of 3×10^{-8} and $1.5 \times 10^{-8}\, \mathfrak{M}_\odot$ year^{-1}, respectively, for α^1 Her. While α^1 Her is intrinsically fainter than α Ori, it is of later spectral type, thus accounting for Weymann's similar rates for the two stars. Nevertheless, the rates are known only to within an order of magnitude.

If a massive star spends an appreciable portion of its helium-burning lifetime as a red supergiant, it may lose some mass and suffer a delay in its transition to a blue supergiant (cf. Hayashi *et al.* 1962). However, its total luminosity (and hence lifetime) would not be significantly affected because its core evolves like a single star. In any case, the amount of mass lost cannot be very great since blue supergiants are observed with masses all the way up to $60\, \mathfrak{M}_\odot$, the limiting mass on the main sequence (Stothers and Simon 1968). After the star becomes a red supergiant again at the onset of carbon burning, further mass loss might be expected to occur. The amount depends on the remaining lifetime, that is, on the existence or non-existence of the neutrino processes, or on any other factor which might affect the red-supergiant phase. However, at least one red supergiant, VV Cep, is known to have a mass as high as the limiting mass on the main sequence (Peery 1966).

b) Rotation

A swiftly rotating star may possess a mixing zone between core and outer envelope, which would develop most strongly when the core is centrally condensed, that is, when hydrogen in the center is exhausted. However, Mestel (1965) has indicated that, unless the star is initially close to breakup, mixing between envelope and core is unlikely to occur. Calculations show that the ratio of centrifugal force to gravity at the core boundary remains about the same throughout the whole subsequent evolution (Sugimoto *et al.* 1968; Stothers and Chin 1969).

c) Duplicity

About 20 per cent of the main-sequence O and B stars are obvious spectroscopic (close) binaries, although a much greater percentage may be undetected wider binaries and multiple stars, especially among the O stars (Jaschek and Jaschek 1957, 1959; Blaauw 1961). Petrie (1960) estimated the percentage of spectroscopic binaries as high as 50 per cent. The fraction of these spectroscopic binaries which are theoretically able to attain directly blue-supergiant and red-supergiant dimensions, respectively, is roughly one-half and one-tenth (Jaschek and Jaschek 1965). Hence, if evolution proceeds from blue to red, 10–30 per cent of the blue supergiants cannot evolve into red supergiants. If, as seems more likely on theoretical grounds, the blue supergiants are in a later evolutionary stage than the "first-stage" red supergiants, then all the red supergiants can evolve into blue supergiants. The latter alternative has some observational support in that (1) the percentage of supergiants showing variable radial velocity (possibly indicating orbital motion) is actually smallest among the bluest supergiants (Jaschek and Jaschek 1957), (2) the variability among the blue supergiants might not be due to binary motion at all (Abt 1957), and (3) according to evidence from the Large Magellanic Cloud, a definite paucity of close binaries occurs among the blue supergiants, as inferred from the remarkable constancy of their light output (Bok 1966) and the monotonic relation between strength of P Cygni characteristics and luminosity (Feast *et al.* 1960).

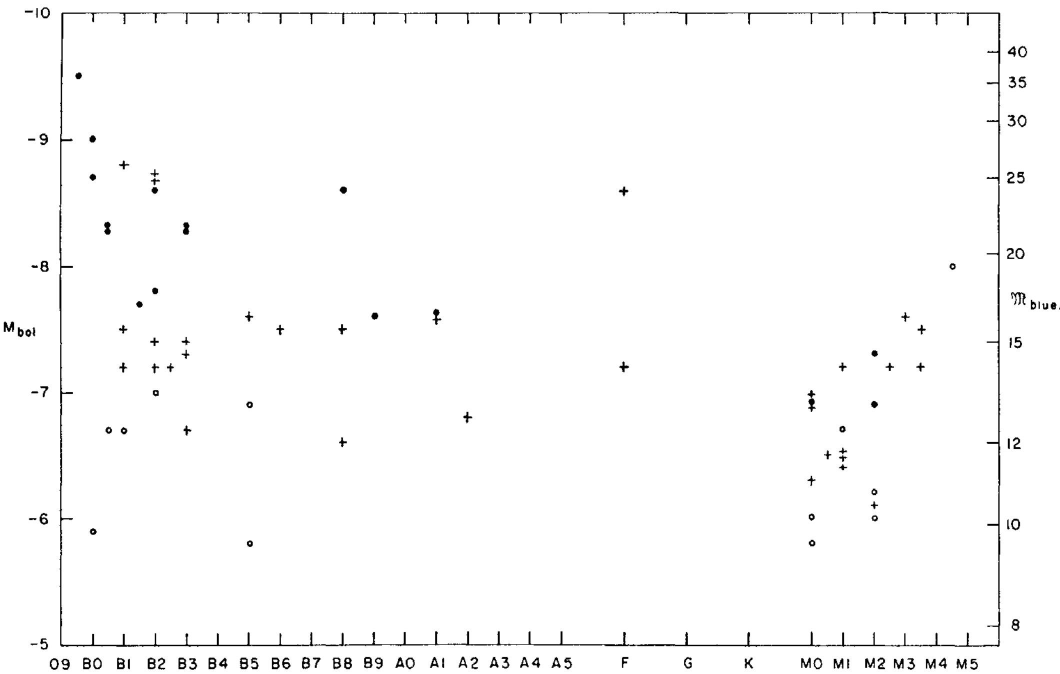

FIG 1.—Bolometric H-R diagram for all known supergiants in galactic clusters and associations containing red-supergiant members. The only omissions are o^1 CMa from Cr 121 and all the stars from the uncertain outer group of I Per. Filled and open circles refer to members of the bright-blue-supergiant and faint-blue-supergiant group of clusters in Table 7, respectively. Crosses refer to members of the other clusters. The scale of masses refers only to the blue supergiants.

We conclude this section by suggesting that mass loss is not an important factor in the evolution of the majority of massive stars. Moreover, effects of rotation and duplicity *in the evolution of virtually all supergiants* are also expected to be unimportant. Consequently, it is meaningful to compare directly numbers of stars in each supergiant region.

VIII. STELLAR STATISTICS AND LIFETIMES

A composite H-R diagram for the supergiants in galactic clusters and associations containing red-supergiant members is shown in Figure 1. The blue supergiants range in spectral type from O9.5 to A5. Roughly half of them are confined to the narrow range B0–B3. A comparable number of red supergiants lies in the range M0–M4.5, but no S or N supergiants are present in our sample. The M spectral type seems to show the latest subdivisions (M2.5–M4.5) among the brightest M supergiants. However, most of the M supergiants lie in the range M0–M2. A definite cutoff in luminosity occurs in the vicinity of $M_{bol} \sim -8$. There is no clear correlation between luminosity class and actual luminosity for the M supergiants. A scattering of yellow supergiants of spectral type

F, G, and K is also found. Although the data are fragmentary, their numbers seem to decrease with advancing spectral type.

The very similar data on field supergiants in the Large Magellanic Cloud supplement and extend our data on galactic clusters and associations to the brightest magnitudes. However, the Cloud sample is very incomplete, especially at late spectral types and at faint magnitudes. It cannot be used for statistical purposes, except for the members of clusters and associations (see below).

In an individual cluster or association, the observed spread in magnitude among the supergiants is often considerable. The scatter is due to several causes: (1) observational errors, (2) cluster back-to-front differences and even non-membership, (3) evolutionary effects, (4) cosmical fluctuations (duplicity, initial chemical composition, rotation, envelope mixing, and mass loss), and (5) variations in stellar times of formation. Observational errors contribute an average spread of about ± 0.3 mag. Uncertainty in the bolometric corrections cause an individual supergiant's bolometric magnitude to be uncertain by ± 0.5 mag but would probably create a systematic trend (not a broad spread) among all the supergiant members considered together. Except in the case of the I Per aggregate, questions of cluster back-to-front differences and non-membership should not be important for our sample. Evolutionary effects may cause an error of ± 0.1 mag. Since single supergiants in the same cluster have approximately the same initial environmental conditions, cosmical fluctuations should be very small. Thus the observed magnitude spread (up to 2 mag in some clusters) must be due to different times of formation of the member stars.

It has recently been found necessary to abandon the supposition of coeval star formation in clusters. A large body of observational and theoretical evidence now exists, which demonstrates that stars in young clusters may be formed over a period comparable, in some cases, with the total lifetime of the most massive stars, namely, 10^6–10^7 yr or even longer (e.g., Westerlund 1961; Wildey 1964; Iben and Talbot 1966; Ezer and Cameron 1967; *passim* in Kerr and Rodgers 1964). Many young clusters seem to have more than one branch (or even a continuum of branches) of stars evolving onto and off the main sequence. Often, the brightest (youngest) blue and red supergiants are found only in the inner regions of a cluster. Since the age of a *blue supergiant* (reckoned from the time of star formation) is directly related to its absolute magnitude by

$$\log \tau \text{ (yr)} = 8.37 + 0.18\, M_{\text{bol}}$$

(on the assumption of a galactic Population I chemical composition), it is possible to date clusters and also to determine the minimum spread of time of formation of their constituent stars by using the observed magnitudes of blue-supergiant members.

To eliminate the effects of a large spread in the mass and time of formation of the supergiants in certain clusters, we have divided the clusters of Table 1 into three groups. The groups are defined on the basis of the absolute magnitudes of the blue-supergiant members of the constituent clusters: all the members have (1) $M_{\text{bol}} < -7.5$, (2) $M_{\text{bol}} > -7.5$, or (3) a scatter around $M_{\text{bol}} = -7.5$. The dividing magnitude is the median magnitude of the blue supergiants in Figure 1 and corresponds to a mass of about 15 $\mathfrak{M}_\odot$. We shall make the assumption that the red supergiants in each cluster lie in approximately the same mass range as the blue supergiants. The clusters and their content of blue, yellow, and red supergiants are listed in Table 7. We have added data for a (typical) cluster of slightly greater age, namely, NGC 6067 (Thackeray, Wesselink, and Harding 1962), which is like the Pleiades group (Eggen 1965) but much richer. It is "typical" in having K supergiants but no B supergiants (cf. Trumpler and Weaver 1953).

A blue/red ratio (number of blue supergiants divided by the number of red supergiants) has been formed for each group of clusters. It represents a *lower limit* to the true (unbiased) blue/red ratio in space, for the following three reasons. First, it is possible that some of the observed red supergiants should not be appropriately included in the ratio because they may be evolved stars of initially much lower mass than their blue counterparts—remnants of an evaporated association around a younger blue cluster. Second, blue giants of luminosity class II have not been included in our sample. Many of these objects are probably also in the helium-burning stage of evolution since they often appear well above, and separated from, the main-sequence turnoff. The following clusters from our sample contain *luminous* blue giants (the number of such objects is given in parentheses): NGC 581 (two), I Per (four), I Ori (one), I Gem (four), Cr 121 (one), NGC 3293 (one), NGC 3766 (three), and IC 2944 (one). Third, there are many well-investigated clusters which apparently do not contain any red supergiants at all and were therefore not included in our list (cf. Morgan, Whitford, and Code 1953; Bidelman 1954). These clusters normally contain rather bright blue supergiants. Consequently, the blue/red ratio will have been underestimated chiefly for the group containing supergiants of highest luminosity.

TABLE 7

LUMINOSITY FUNCTION OF SUPERGIANTS IN GALACTIC CLUSTERS AND ASSOCIATIONS WITH RED-SUPERGIANT MEMBERS

M_{bol} (Blue)	$\mathfrak{M}/\mathfrak{M}_\odot$	n_b (Blue)	n_y (Yellow)	n_r (Red)	n_b/n_r	Clusters
−7 6 to −9 5	15–35	13	0	3	4 3	h Per, I Ori, NGC 3293, NGC 4755
−5.6 to −7.5	10–15	6	0	6	1 0	NGC 581, χ Per, NGC 3766, IC 2944, II Sco
(−3)	5	0	3	8	0	NGC 6067
−5 6 to −9 5	10–35	7	3	5	1 4	NGC 457, I Gem, Cr 121
−5 6 to −9 5	10–35	11	0	9	1.2	I Per (inner), χ Per
−5 6 to −9.5	10–35	16	5?	9	1 8	I Per (outer), h Per

Despite the rather small number of stars at our disposal, a clear trend emerges from Table 7: as the luminosity increases, the blue/red ratio becomes larger. In view of our previous remarks, this ratio must be considerably greater than 5 in the case of the bright group of clusters. Supporting evidence is indicated by the trend in I Per. Nearly 60 per cent of the blue supergiants in the outer group of I Per have "high" luminosity, whereas only 20 per cent in the inner group of I Per do; it is clear that the blue/red ratios for the two groups in I Per approximately reflect these distributions.

In the Magellanic Clouds the situation is analogous. Areas and clusters which contain stars of moderate luminosity show a much lower blue/red ratio than do typically "blue" areas with stars of higher luminosity. Luminosity functions have been determined for a number of young clusters and associations in the Large Cloud. We have used the published color-magnitude diagrams due to Westerlund (1961) and to Woolley (1960) for a total of fourteen clusters and associations. Although spectroscopic data are generally lacking, we have assumed that the clumps of blue and red stars lying well above and to the right of the main sequence, respectively, are indeed supergiants. Bright yellow stars were ignored as mostly being foreground stars in our own Galaxy (Woolley and Epps 1965). The results are presented in Table 8, where we have arranged the clusters in order of increasing faintness of the main-sequence turnoff. Despite the larger uncertainty in the star numbers here compared with the star numbers in our own Galaxy, the results for the Large Cloud are in good agreement with the results for our Galaxy even if no account is taken of the fact that a few of the red stars may be foreground objects superimposed on the Cloud.

The relative lifetimes of the various classes of supergiants can now be obtained directly from their relative star numbers. While such a procedure may not be valid in the case of an *individual* cluster where the birth-rate function can vary drastically and irregularly over a relative small mass interval, it is certainly valid in the case of a *group* of clusters where any irregularities in the initial distribution of masses will be smoothed out.

The observational blue/red ratio for stars in the mass range 15–35 $\mathfrak{M}_\odot$ is >5 (the median mass in our sample is 22 $\mathfrak{M}_\odot$). If we assume that the blue supergiants are in the core helium-burning phase of evolution and that all the red supergiants are in the core-contraction (helium and carbon) and late phases, then the theoretical models without neutrino emission predict a *maximum* blue/red ratio (ratio of lifetimes) equal to 1.7. If

TABLE 8

LUMINOSITY FUNCTION OF SUPERGIANTS IN CLUSTERS AND ASSOCIATIONS OF THE LARGE MAGELLANIC CLOUD

M_v (Turnoff)	n_b (Blue)	n_r (Red)	n_b/n_r	Clusters
−7	3	1	3 0	NGC 1962–5–6–70
−6	10	10	1 0	NGC 1984, NGC 1994, NGC 2074, NGC 2081, Anon b2, Anon b3, Anon b4
−5	18	25	0.7	NGC 1818, NGC 2004, NGC 2011, NGC 2100
−4	0	4	0	NGC 1810, NGC 2092

some of the red supergiants are actually in the core helium-burning phase, then the predicted blue/red ratio is <1.7. In our present sample of clusters, where we observe thirteen blue and three red supergiants (remembering that we have selected only clusters containing at least one red supergiant), we should expect *at the very least* nine red supergiants!

With the inclusion of neutrino processes in the theoretical models, however, the predicted blue/red ratio is ~ 17 if core helium burning takes place only (or mainly) in the blue region of the H-R diagram. This is more than adequate to explain the relative superabundance of massive blue supergiants. If the actual blue/red ratio in space happens to be less than 17 (our data yield only >5), some of the massive red supergiants must be in the core helium-burning phase. In the group of supergiants with lower mass, the smaller observed blue/red ratio (~ 1) must be due to the fact that stars of lower mass spend a greater fraction of their helium-burning lifetime in the red region.

IX. CONCLUSION

The status of the red supergiants as a class can now be fairly well understood. The clearest evidence is drawn from the data on the supergiants in well-observed clusters and associations. The clusters with the brightest blue main-sequence stars usually lack both blue and red supergiants altogether. Those with the brightest blue supergiants contain very few red supergiants, while those with many relatively faint red supergiants are found entirely lacking in blue supergiants. Theoretically interpreted, this arrangement of the clusters by morphology of their H-R diagrams is an arrangement in order of advancing evolutionary age (decreasing mass of the stars at the main-sequence turnoff).

Deutsch's (1956*a*) inference that the M supergiants must be evolutionarily old is supported by our results. Just as in the case of the K giants (Sandage 1957), there is an evolutionary "funneling effect" into (and out of) the region of M supergiants on the

H-R diagram. In the present case, however, the funnel is tilted down rather than up, having an upper boundary (red) at $M_{bol} \sim -8$. Most of the M supergiants lie in the spectral interval M0–M2, although the brightest ones seem to have the latest spectral type, ranging from M2.5 to M4.5.

It is usually impossible to determine the evolutionary phase attained by an *individual* red supergiant. The star may be in a helium core phase or in a carbon core (or later) phase. Although a one-to-one correspondence exists between luminosity and mass in the case of the blue supergiants, such a correspondence is lacking for the red supergiants because of the observational and theoretical funneling effect.

When the data are treated statistically, certain conclusions may be drawn concerning the red supergiants On the basis of the large blue/red ratio among the most massive group of supergiants and the independent theoretical and observational evidence that early core helium burning occurs in the red-supergiant region, we conclude that some effect must take massive stars very rapidly through, or away from, the red-supergiant region after core helium burning. Complete mixing of the stars has been shown not to occur. Nor is mass loss expected to be effective since the total mass must apparently remain virtually unaffected during the *previous* red-supergiant phase (early core helium burning) which lasts at least a comparably long time as the later phase in most of the supergiants.

It is suggested that massive stars do evolve *as red supergiants* (but very rapidly) after core helium burning. Such an acceleration of evolution is amply provided by photoneutrino energy losses. Thus, the average neutrino "luminosity" would be at least an order of magnitude greater than the optical luminosity during the most advanced phases.

If this interpretation of the observational data is correct, we may then be able to infer the actual existence of a direct electron-neutrino weak interaction in nature. It is hoped that future work may provide additional information particularly on the statistics of supergiants in very young clusters and associations as well as on the theoretical evolution of massive stars during early core helium burning.

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